E 1546 (Pages : 4) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, APRIL 2018

Fourth Semester

Core Course—VECTOR CALCULUS, THEORY OF EQUATIONS AND NUMERICAL METHODS

(Common for B.Sc. Mathematics Model I, II and B.Sc. Computer Applications)

[2013 Admission onwards]

Time : Three Hours Maximum Marks : 80

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Write the parametric representation of the sphere $x^2 + y^2 + z^2 = a^2$.
2. What is the component of the vector $2\hat{i} + 0.5\hat{j} + 0.3\hat{k}$ in the direction of $0.2\hat{i} + 1.4\hat{j} + 3\hat{k}$?
3. Find the gradient of the function $g(x, y) = y - x$ at the point $(2, 1)$.
4. State fundamental theorem of Algebra.
5. Is zero a root of $x^5 - 5x + 2 = 0$? Justify.
6. State Gauss's Divergence theorem.
7. Describe Cardan's method.
8. Find the divergence of $\bar{r}$.
9. State Remainder theorem.
10. State the formula for the method of false position.

(10 × 1 = 10)

Part B

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. If f is a differentiable function and ϕ is a differentiable scalar function , then prove that $div(\phi f) = \phi\, div\, f + f.(grad\ \phi)$.

Turn over

12. Find the directional derivative of $2xy + z^2$ in the direction of the vector $i + 2j + 2k$ at the point $(1, -1, 3)$.

13. If $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$, prove that $\nabla^2 r^n \bar{r} = n(n+3) r^{n-2} \bar{r}$.

14. Prove that $\nabla\left(\frac{f}{g}\right) = \frac{g\nabla f - f\nabla g}{g^2}$.

15. Find the tangent plane and normal line of the surface $(x, y, z) = x^2 + y^2 + z - 9 = 0$ at the point $(1, 2, 4)$.

16. Find the equation for the tangent plane to the surface $x^2 + y^2 + z^2 = 3$ at the point $(1, 1, 1)$.

17. If α, β, γ are the roots of the equation $x^3 + qx + r = 0$ then find (i) $\sum \frac{1}{\beta + \gamma}$, (ii) $\sum \frac{1}{(\beta - \gamma)^2}$.

18. Solve $2x^3 - x^2 - 18x + 9 = 0$, if two of the roots are equal in magnitude but opposite in sign.

19. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the equation whose roots are $\alpha\beta + \beta\gamma, \beta\gamma + \gamma\alpha, \gamma\alpha + \alpha\beta$.

20. Solve the reciprocal equation $6x^4 + 35x^3 + 62x^2 + 35x + 6 = 0$.

21. Find a root of the equation $x^3 - 4x - 9 = 0$ correct to three decimal places by using bisection method.

22. Using Newton-Raphson formula, establish the iterative formula $x_{n+1} = \frac{1}{2}\left[x_n + \frac{N}{x_n}\right]$ to calculate the positive square root of N.

(8 × 2 = 16)

Part C

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. Verify Gauss Divergence theorem for $F = 4x\bar{i} - 2y^2\bar{j} + z^2\bar{k}$ taken over the region bounded by $x^2 + y^2 = 4, z = 0, z = 3$.

24. Find $\int_C x^2\, y\, dx + ydy$, where C is the closed curve formed by $y^2 = x$ and $y = x$ between $(0, 0)$ and $(1, 1)$ using Green's Theorem.

25. Find the work done by $F = (y - x^2)\hat{i} + (z - y^2)\hat{j} + (x - z^2)\hat{k}$ over the curve $r(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k}\ 0 \le t \le 1$ from $(0, 0, 0)$ to $(1, 1, 1)$.

26. Find the flux of $F = (x - y)\hat{i} + x\hat{j}$ across the circle $x^2 + y^2 = 1$ in the xy-plane.

27. Solve by Cardan's method : $x^3 - 18x - 35 = 0$.

28. Find the number and position of the real roots of the equation $x^3 - 3x + 1 = 0$.

29. If α, β, γ are the roots of the equation $x^3 - 6x^2 + 11x + k = 0$ find the equation whose roots are $\alpha - 2\beta + \gamma, \beta - 2\gamma + \alpha, \gamma - 2\alpha + \beta$. Also find k so that the roots are in AP.

30. Find an approximate value of the root of the equation $x^3 + x - 1 = 0$ near $x = 1$, by the method of false position.

31. Using Newton-Raphson method, find correct to four decimals the root of the equation $\sin x = 1 + x^3$ lies between – 2 and – 1.

$(6 \times 4 = 24)$

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Verify Stokes theorem for $F = (x^2 + y^2)\hat{i} + -2xy\,\hat{j}$ taken around the rectangle bounded by $x = \pm a, y = 0, y = b$.

(b) If $r = |\bar{r}|$ where $\bar{r} = xi + yj + zk$ then prove that $\nabla\frac{1}{r} = -\frac{1}{r^3}\bar{r}$.

33. (a) Find the constants a, b, c so that the vector

$\bar{f} = (x + 2y + az)\bar{i} + (bx - 3y - z)\bar{j} + (4x + cy + 2z)\bar{k}$ is irrotational.

(b) Verify Gauss's Divergence Theorem, for $\bar{F} = (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^3 - xy)\bar{k}$ over the rectangular parallelopiped $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$.

34. (a) Solve by Ferrari's method : $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$.

(b) Solve $x^3 - 9x^2 + 14x + 24 = 0$, two of whose roots being in the ratio $3 : 2$.

35. (a) Find the root of the equation $x^3 - x - 11 = 0$, using bisection method correct to three decimal places which lies between 2 and 3.

(b) Find a positive root of $x^2 + 2x - 2 = 0$, by Newton-Raphson method, correct to two significant figures.

$(2 \times 15 = 30)$